

# Shoplifting Prediction Using Computer Vision: Spatio-Temporal Video Abstraction via Probabilistic Graphs

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**Abstract.** Anomaly detection in video surveillance, such as shoplifting, faces significant challenges due to high tensorial dimensionality, sensitivity to visual noise and highly computational costs. This paper proposes an approach that abstracts video sequences into directed probabilistic graphs. Through kinematic pose extraction and stochastic modeling of micro-states, the system translates complex spatio-temporal dynamics into topological signatures. We introduce a Structural Affinity metric  $D_t$  to evaluate network divergence in real-time. Experimental validation on a standardized dataset demonstrates that routine shopping behavior exhibits a dense, cyclic topology, whereas illicit actions break this pattern. Our approach ensures clear structural separability, providing a robust, interpretable, and computationally low-cost analytical foundation for automated security systems.

**Keywords:** Computer vision, probabilistic graphs, shoplifting detection, topological abstraction, spatio-temporal representation.

## Predicción de hurtos en tiendas mediante visión artificial: abstracción espacio-temporal de vídeo a través de grafos probabilísticos

**Resumen.** La detección de anomalías en videovigilancia, como el robo en tiendas, presenta desafíos importantes debido a la elevada dimensionalidad tensorial, la sensibilidad al ruido visual y los altos costos computacionales. Este artículo propone un enfoque que abstrae secuencias de video en grafos probabilísticos dirigidos. Mediante la extracción cinemática de la postura y el modelado estocástico de microestados, el sistema traduce dinámicas espacio-temporales complejas en firmas topológicas. Introducimos una métrica de afinidad estructural,  $D_t$ , para evaluar la divergencia de la red en tiempo real. La validación experimental en un conjunto de datos estandarizado demuestra que el comportamiento habitual de compra exhibe una topología densa y cíclica, mientras que las acciones ilícitas rompen este patrón. Nuestro enfoque garantiza una clara separabilidad estructural, proporcionando una base analítica robusta,

interpretable y de bajo costo computacional para sistemas de seguridad automatizados.

**Palabras clave:** Visión por computadora, grafos probabilísticos, detección de robo en tiendas, abstracción topológica, representación espacio-temporal.

## 1 Introduction

The increased development and adoption of video surveillance systems have led to the generation of enormous volumes of visual data [1]. Real-time analysis of video information for anomaly detection is one of the most prominent tasks currently underway in the field of computer vision. Within this task, identifying criminal acts such as shoplifting, which involves subtle actions easily mistaken for common behavior, presents a complex problem. In contrast to anomalies such as assaults or traffic accidents, shoplifting typically involves joint interactions between the perpetrator and their surroundings, characterized by concealment or reconnaissance of the scene where the events take place [2], and which visually resemble the routine behavior of ordinary customers.

In recent years, researchers have approached video analysis by processing entire video sequences using tensors in multidimensional spaces, where spatial and temporal features are processed together, and by implementing complex deep learning architectures such as three-dimensional convolutional neural networks (3DCNNs). While these approaches demonstrate highly accurate detection rates, they also have significant limitations, such as considerably large computational costs, sensitivity to environmental variations like noise, blur, or changes in lighting, and a lack of explainability resulting from their black-box construction, which hinders the understanding and selection of the characteristics of an action that might trigger a burglary alert.

To address these weaknesses, this paper proposes an approach focused on the structural representation of the thief's behavior. Instead of delegating the extraction of temporal features to an architecture, an explicit model of the sequence of body movements is designed in a probabilistic directed graph.

This proposal argues that dynamic behaviors can be discretized into a space of finite semantic states or nodes, where the development of actions is modeled through transitions or edges. By translating the sequence of raw human poses into a graph topology, three contributions are achieved: 1) Drastic reduction of dimensionality by transforming thousands of similar human poses into a compact adjacency matrix. 2) Extraction of the topological signature of theft demonstrating that the anomalous behavior associated with shoplifting exhibits structures different from those related to normal activity. 3) Robust description of visual disturbances, establishing mathematical foundations for classification models to be developed in the future.

The remainder of this paper is organized as follows: Section 2 reviews the state of the art in graph-based video representation and anomalies. Section 3 details the mathematical formulation of graph representation, followed by Section 4, which presents experimental results. Finally, Section 5 sets out the conclusions and guidelines for future work.

## 2 Related Work

The computational representation of video events has been approached from multiple perspectives, ranging from extracting local frame features in the spatial domain to modeling global relationships by considering the contextual and temporal information of a series of frames.

For years, action recognition relied heavily on manually extracted features. Wang et al. [3] developed a dense path-based approach that tracked key points in video using optical flow. Their method extracted local descriptors, such as oriented gradient histograms (HOGs), optical flow histograms (HOFs), and motion boundary histograms (MBHs). While still an effective method for capturing motion patterns in short actions, it lacks a global description of the classes. Consequently, it cannot capture the semantics of long-term actions or the dependencies needed to identify prolonged and complex events such as shoplifting.

More recently, graph-based representation has become increasingly common for capturing spatiotemporal dependencies, popularized primarily by the rise of deep learning. In [4], Yan et al. introduced Spacetime Graph Convolutional Networks (ST-GCNs). Their method constructs graphs from human skeletons extracted using pose estimation, where joints serve as nodes, and identical joints in consecutive frames form temporal edges. Although ST-GCNs efficiently learn spatial and temporal patterns, they treat the graph simply as a data structure for deep feature extraction. Similarly, Wang and Gupta [5] proposed representing videos as Spacetime Region Graphs by extracting bounding boxes from objects and computed features with I3D (Inflated 3D ConvNet). They connect objects and actors as nodes based on appearance similarity and spatial proximity to capture interactions through Graph Convolutional Networks (GCNs).

While highly accurate, both approaches [4, 5] rely on massive and computationally expensive deep architectures that operate on continuous streams of pixels or coordinates, effectively obscuring the meaning of behavior within black boxes.

The community has also focused on capturing explicit relationships and mitigating noise in surveillance videos. Chen et al. [6] addressed anomaly detection using Spatiotemporal Attention Graph Networks (STGAs). By extracting skeletons and representing them as graphs, attention scores are calculated to adaptively assign weights to each joint, determining actions and mitigating background disturbances, thus allowing for the explicit modeling of relationships between objects and the scene.

Zhang et al. [7] proposed a Cognitively Guided Video Anomaly Detection (CG-VAD) framework that integrates scene knowledge graphs with Convolutional Neural Networks (CNNs) to evaluate object interactions. However, despite their effectiveness in filtering noise, these methods still rely on complex attention and continuous processing mechanisms, requiring significant computational resources that limit their scalability for real-time commercial surveillance with multiple data streams.

Despite the advances made in graph theory in recent years, the current trend focuses on generating graphs directly from deep neural networks or using them as intermediate structures for graph convolutions. This approach seeks to maximize accuracy at the expense of transparency and interpretability.

This work departs from this trend. It does not aim to use graphs as hidden intermediate layers within a deep architecture, nor to process continuous convolutional

features. Instead, it translates movement data into discrete microstates to construct a probabilistic transition graph. This graph acts as a direct and interpretable representation of the events occurring at the scene, enabling the analysis and understanding of the thief's behavior through topological divergence.

### 3 Methodology

The core of the approach proposed in this article consists of transforming a video sequence into a directed probabilistic graph. Since visual features can be sensitive to noise, human behavior is abstracted into the structural domain, following a stochastic model.

#### 3.1 Dataset Characteristics

The primary attributes of the MNNIT Shoplifting Dataset [2] are detailed in Table 1. The dataset consists of short video sequences captured in a controlled laboratory environment simulating a retail store.

Unlike standard deep learning architectures that require a strict train and test data split to optimize weights, our proposed graph-based framework focuses on extracting a mathematical topological representation. The sequences were manually tagged with 28 semantic microstates. This evaluation ensures that the statistical distribution and the optimal separability threshold are representative of all available behavioral variations without the bias of a restricted testing partition.

#### 3.2 Extraction and Preprocessing

Given a video sequence, the initial phase focuses on isolating the thief's dynamics through a structured preprocessing pipeline, as illustrated in Figure 1.

To ensure robustness before graph construction, the raw data undergoes the following stages:

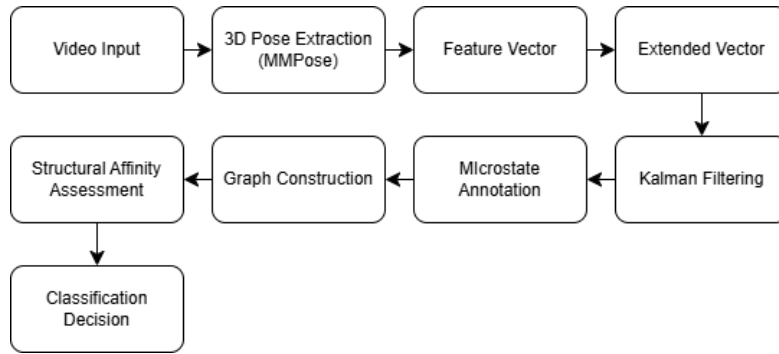
**3D Pose Extraction:** Starting with the video input, skeletons are extracted using the MMPose estimation model [8] to determine the three-dimensional spatial coordinates of the human body's joints.

**Kinematic Feature Vector Construction:** From these coordinates, a 15-dimensional vector containing selected biomechanical angles is calculated. Specifically, these 15 features were deterministically selected to focus on upper-body kinematics (e.g., shoulder flexion, elbow extension, wrist rotation, and torso inclination). This criteria intentionally isolates the manipulative actions essential to shoplifting and concealment, effectively filtering out irrelevant ambulatory noise from the lower limbs. To fully capture the temporal dynamics of the actions, this vector is expanded to 45 dimensions by computing and appending the angular velocity and angular acceleration for each angle at every frame.

**Noise Reduction and Smoothing:** Since raw video data and pose estimations are highly susceptible to visual noise, lighting variations, or occlusion, a Kalman filter [9]

**Table 1.** Key characteristics of the MNNIT Shoplifting Dataset.

| Attribute            | Specification                                                                                |
|----------------------|----------------------------------------------------------------------------------------------|
| Recording conditions | Controlled indoor setup, 32MPX camera                                                        |
| Resolution           | 620x480 pixels, (5 videos with 1920x1080 pixels)                                             |
| Frame Rate           | 30 frames per second                                                                         |
| Total Sequences      | 183 clips, around 300 frames (10 seconds) per clip                                           |
| Class distribution   | 93 Shoplifting sequences, 90 Normal behavior sequences                                       |
| Action diversity     | <b>Normal:</b> walking, inspecting items.<br><b>Shoplifting:</b> concealment in clothes/bags |

**Fig. 1.** Proposed general methodology pipeline detailing the transformation of raw video input into topological signatures for anomaly classification.

is applied to the 45-dimensional sequences to obtain clean and smooth kinematic trajectories prior to their discretization.

To discretize the continuous space of body movements, the graph nodes  $V$  are defined by 28 semantic microstates corresponding to movements performed by the person e.g., 'approaching shelf', 'standing', 'reaching forward', 'inspecting item', 'retracting arm', 'concealing in pocket', etc.

This taxonomy was derived empirically from observing the dataset to capture the sequential intent behind the motion. Ground Truth is implemented through supervised annotation in which the training videos were analyzed using a sampling interval of every 5 frames, as recommended for motion capture [10]. At each selected time point, a label is assigned to the action performed or the state  $v_k$  corresponding to the present action. This systematic sampling technique reduces the redundancy inherent in a 30fps video and ensures that each node  $v_k$  represents a specific action intention and not a fleeting movement.

### 3.3 Graph Construction

The jump between actions that determine criminal activity is modeled as graph transitions. The adjacency matrix is constructed by estimating the conditional

probability of transitioning from state  $v_i$  to state  $v_j$ . Based on manually determined sequences, the transition probability is estimated following the Maximum Likelihood Estimation (MLE) principle for Markov chains [11] following Equation 1:

$$P(v_j|v_i) = \frac{C(v_i \rightarrow v_j)}{\sum_{k \in V} C(v_i \rightarrow v_k)}, \quad (1)$$

where  $C(v_i \rightarrow v_j)$  is the number of transitions in the adjacency matrix constructed from the video set from state  $v_i$  to state  $v_j$ , and  $v$  is a state from graph. However, for sparse graphs, unannotated or very infrequent transitions can generate probability of zero, due to the computational nature of the calculations. To mitigate this condition, Laplace Smoothing [12] is applied, transforming the matrix into a logarithmic space by introducing a regularization term  $\epsilon = 1 \times 10^{-20}$ . This step prevents the arithmetic underflow that can occur in long strings of probabilities close to zero, as can be seen in Equation 2:

$$\log P(v_j|v_i) = \log P(v_j|v_i) + \epsilon. \quad (2)$$

### 3.4 Temporal Modulation

A traditional Markov chain is characterized by memory loss; that is, the probability of leaving a state does not depend on the time spent on it. However, human behavior can be influenced by the length of time spent performing an action. To model this characteristic, the graph explicitly penalizes remaining in the same state [13].

It is defined  $d$  as the duration the offender remains in the current state  $v_i$ , and a time modulation function is introduced. The logarithmic probability of remaining in the same state is dynamically penalized at each instant in time based on a decay rate  $\lambda$  according to Equation 3:

$$\log P'(v_j|v_i) = \log P(v_j|v_i) - \lambda(d + 1). \quad (3)$$

This formula emulates exponential decay and is based on the modeling of state duration and decay distributions [14]. The value  $\lambda$  forces the graph to diminish the importance of the current action after a reasonable time, allowing the system to progress through the action graph. By explicitly penalizing state retention based on duration, the proposed framework transitions from a strict Markovian assumption into a Semi-Markovian space. This mathematical modulation acts as a temporal memory mechanism, satisfying the long-range dependencies required by complex human behaviors without the prohibitive computational overhead of deep recurrent architectures.

### 3.5 Structural Affinity

To classify the topology observed during the playback of a video sequence, the system simultaneously maintains two matrices  $P_{normal}$  and  $P_{shoplifting}$ . When a transition occurs ( $v_i \rightarrow v_j$ ), the jump divergence is calculated using an approximation of the Likelihood Ratio test [15] adapted to measure a Structural Affinity that allows the use



**Fig. 2.** (a) Frames corresponding to normal behavior. (b) Frames showing shoplifting. Images from MNNIT Shoplifting Dataset [2].

of both graphs, normal and shoplifting. The level of theft alarm is calculated based on Equation 4:

$$A_{trans} = \left( \frac{P_{shoplifting}(v_j|v_i)}{P_{shoplifting}(v_j|v_i) + P_{normal}(v_j|v_i) + \epsilon} \right) \times 100. \quad (4)$$

Since video sequences can contain noise caused by atypical micro-movements lasting only an instant, evaluating a single frame can produce unstable results. Therefore, instantaneous affinity should be combined with a global metric  $D_t$  using a Weighted Moving Average (EWMA) [15], see Equation 5:

$$D_t = \alpha D_{t-1} + (1 - \alpha) A_{trans}, \quad (5)$$

where  $\alpha \in [0,1]$  behaves as a historical weighting or inertia factor inspired by time series analysis for quality control [16].

## 4 Results

To validate the topological mapping capability of the behavioral patterns, the entire dataset was processed, considering all 93 shoplifting sequences and the 90 normal behavior sequences. The main objective of these experiments was to evaluate the effectiveness of data representation and to depict the structural separability between the two classes from a mathematical and graphical perspective before processing the data with any type of classifier.

The central proposition of this paper establishes that licit and illicit behaviors possess divergent topological signatures. To verify this, graph metrics were extracted for each video in the dataset. In the Scatter Plot of Figure 3 shows the clustering coefficients against the graph density of each video evaluated. The  $x$ -axis correspond to the clustering coefficient, which quantifies the formation of cyclical and return sequences, where values tending towards zero reveal a certain acyclic directionality. Simultaneously, the  $y$ -axis represents the network density, which measures the

proportion of transitions executed, that is, the complexity of the chain of actions and gives indications of a clear or random intention.

Normal sequences tend to be located in the middle and upper regions of both axes. A high clustering coefficient indicates a greater prominence of cyclical behaviors, such as picking up objects and observing them out of curiosity rather than performing a specific action. Higher density also reflects action sequences consistent with less restricted activity. A person who does not engage in shoplifting tends to perform closed cycles, such as picking up a product, examining it, and returning it to repeat the process. Therefore, the network of actions in normal behaviors tends to be more convoluted, as the chain of movements is often more erratic and random.

Shoplifting sequences maintain lower values in both density and clustering coefficient, indicating predominantly linear rather than cyclical behaviors, and suggesting that thefts are represented by less complex behaviors. The low clustering coefficient prevalent in shoplifting examples suggests patterns that move primarily in one direction with more sporadic returns or cycles compared to normal behaviors. Furthermore, the lower density compared to sequences without shoplifting is consistent with premeditated actions and a clearer objective, ignoring other possible actions.

Although the decision boundary is not obvious, the point clouds of both classes are positioned in particular regions, indicating that this kinematic discretization is capable of effectively isolating anomalies.

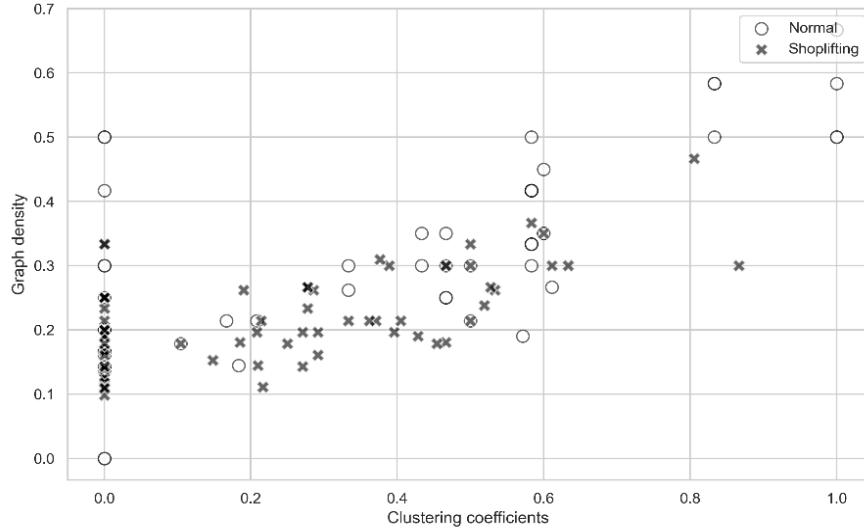
To quantify the system's response to theft, the maximum structural affinity, or theft probability, reached by each video was evaluated. The maximum values are summarized in Figure 4. The results suggest that, statistically, both types of behavior are completely separable.

Statistical analysis of the  $D_t$  distribution shows that for normal behavior, the median divergence is 44%, with a range from 41% to 48%. The low variance and compact dispersion indicate that most legitimate sequences remain similar and without significant deviations. It is important to mention that the absolute maximum is 59%, an observation that helps establish a divergence decision threshold, as will be discussed in the following paragraph.

Conversely, the shoplifting distribution exhibits a median that spikes to 87%, with a range between 82% and 93%. While the dispersion is greater, a consequence of the variations in object concealment techniques, the concentration of most instances in a block of no more than 10% indicates that the method captures the majority of illicit action sequences. Even the outliers present at 61% divergence allow for setting a strict threshold of 60% optimally to separate the two classes, as the total number of normal sequences remains below the threshold, indicating a low rate of topological false positives. Therefore, ambiguous movements do not appear to have a significant impact on the model. Furthermore, the distribution of shoplifting videos shows that most exceed 80% divergence, unlike normal videos, which remain below 50%.

As a particular case to illustrate the behavior of structural divergence in real time, Figure 5 shows the evolution of this metric over time for all videos in dataset.

Figure 5 shows the temporal evolution of structural divergence through transitions in the video sequences. It should be noted that, to standardize sizes, all sequences in each class are adapted to the largest video size, maintaining the divergence value over extended time intervals. When analyzing the behavior in the normal class, values below



**Fig. 3.** Scatter Plot of separability of classes considering clustering coefficients and graph density for all dataset.

the threshold are observed at all times. While the maximum ranges are variable, the flatness is stable, remaining at the bottom of the graph.

In contrast, the trajectories associated with shoplifting tend to increase abruptly, and once they exceed 60% divergence, the risk rarely falls below this threshold again. The fact that videos of this class cluster above videos of innocent activity demonstrate the model's ability to profile anomalous behaviors associated with theft with a low false positive rate.

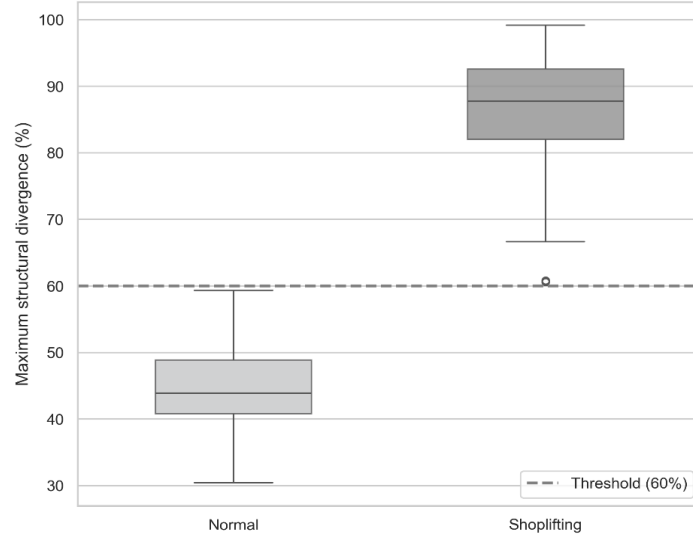
Experiments on the dataset show that criminal actions possess a distinct and isolable structural signature characterized by low density and a low clustering coefficient, behavior contrary to normal sequences.

It has also been found that formulating a structural divergence metric that takes past actions into account allows for reliably stable reaction to and recognition of anomalies in the face of micro-movements in noise. Finally, graph representation drastically reduces data complexity by using adjacency matrices, enabling the analysis of multiple surveillance streams with minimal computational resources.

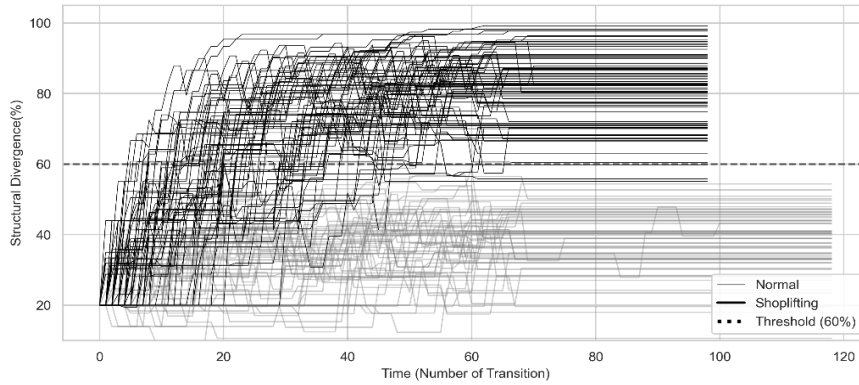
The temporal decay ( $\lambda$ ) and EWMA inertia ( $\alpha$ ) parameters were determined through a sensitivity analysis exploring the range [0.1, 0.9] for both variables. For the decay parameter, values above 0.7 caused the system to retain memory of previous micro-states for too long, delaying the transition into anomaly detection.

Conversely, values below 0.3 forced the model to forget past actions too quickly, making it overly sensitive to noise. The balance was found at  $\lambda = 0.5$ , which aligns with the natural duration of human reaching and concealing motions.

Similarly, for the inertia factor, an  $\alpha < 0.5$  reduced the historical weighting, acting almost like an instantaneous derivative that triggered false positives due to minor pose estimation inaccuracies.



**Fig. 4.** Box Plot of distribution of the maximum peak of structural divergence  $D_t$  reached by each evaluated video.



**Fig. 5.** Temporal evolution of the structural divergence  $D_t$  evaluated on the complete dataset.

Values of  $\alpha > 0.8$  heavily suppressed sudden topological divergences, creating false negatives by ignoring legitimate theft actions. Establishing  $\alpha = 0.7$  successfully smoothed tracking jitter while maintaining a swift reaction to actual illicit structural signatures.

## 5 Conclusions

This article presents a framework for modeling complex human behavior in video sequences using topological abstraction. By discretizing motion data into probabilistic

graphs, the proposed methodology captures the spatiotemporal dynamics of an event without the computational exhaustiveness associated with continuous frame-by-frame pixel analysis.

Translating sequences of human postures into microstates achieves dimensionality reduction by using lightweight adjacency matrices, preserving the behavioral intention of the shopper or thief. Experimental results show that shoplifting actions possess a distinctive and isolable structural signature.

This work shown that anomalous events tend to deviate from the cyclical and exploratory nature of normal shopping behavior. Instead of forming dense networks with high clustering coefficients, shoplifting sequences choose directed and acyclic paths characterized by low graph density and a terminal focus on concealment. Furthermore, the implementation of the structural affinity metric  $D_t$  proved effective in reliably tracking and isolating anomalies, even considering the variability and noise of benign micromovements.

Finally, this approach indicates that structural divergence works as a reliable and practical indicator of shoplifting intent. The observed separability between legitimate and illicit trajectories can yield a low false positive rate, which is an essential requirement for real-world security implementations.

Future research will focus on transitioning this framework from a descriptive anomaly detection tool to a fully predictive environment. The proposed path involves using these graph-based features as direct inputs for advanced machine learning architectures.

By combining this structural representation with continuous estimators of human posture, the final objective is to establish a real-time anticipated probability of shoplifting, enabling intervention strategies in busy retail environments.

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